



Fig. 3 Supply output waveforms

and the second was with the power supply operating in series with a battery bias. Figures 3a and 3b compare the power source output for these operational modes. Engine operation with the power supply and battery combination offers higher overall efficiency and reduces the heat dissipation requirements of the power supply.

Two basic engine designs, one with fixed electrodes and the other with pneumatically operated movable electrodes, have been started and operated with the power supply. The fixed-electrode engine required an externally applied high voltage to ionize the gaseous fuel and to reduce the arc resistance sufficiently to sustain conduction. One means of applying the starting voltage was to introduce a charged capacitor across the electrodes.

Starting tests began by determining the operating characteristics of the engine and power supply separately.

High voltage starting tests were performed with both hot and cold engines. Approximately 2000 v were required to start a cold engine. Typical steady-state engine operating voltages and currents using electric starting were 1) 110 v at 9.7 amp, 2) 103 v at 9.5 amp, and 3) 100 v at 10.0 amp.

### Conclusions

Apparent advantages and disadvantages of the high voltage starting techniques still are being evaluated. Operational disadvantages encountered were 1) requirement of high voltage components and insulation, 2) detrimental influence of high voltage on engine life, and 3) need to adjust starting voltage (800 to 2000 v) to compensate for engine temperature.

Development of a mechanical start mechanism is a noteworthy advancement in the design of the arc engine. Starting of engines with movable electrodes could be accomplished at lower current levels and appeared to be more positive. This method was also more compatible with the power supply and eliminated the need for an additional high voltage source. The desirability of a constant current power supply in this case results in the fact that it also can limit the short circuit current at starting. Because of the initial short circuit with mechanical starting, a power supply and battery bias system requires additional current-limiting equipment. An external current-limiting resistor was provided for this type of start. In addition, a means also was provided for removing the limiter when the engine approached steady-state operation.

### Summary

In summary, the advanced prototypes of the 1-kw arc engine and the power supply performed satisfactorily under laboratory test conditions. Although extensive testing still is to be performed, the power supply is considered to be practical for flight use. In addition to many operational advantages, the power supply offers significant savings in efficiency and payload weight. The degree of arc stabilization and control obtained with the saturable reactor is particularly attractive.

Since the saturable reactor can be operated directly from an a.c. source, it shows even more promise for future applications involving larger engines and a.c. power sources such as a SNAP VIII alternator.

## Criterion for Slip near the Leading Edge of a Flat Plate in Hypersonic Flow

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On the basis of strong interaction theory estimates, a rarefaction parameter of the form  $M_\infty^2 C_\infty / Re_\infty$  is proposed which appears to be useful in correlating experimental data in the slip-flow region near the leading edge of a flat plate. This parameter, which in fact is the square of the low-speed rarefaction parameter, can be interpreted as a measure of both the ratio of the mean free path behind the shock wave to the boundary layer displacement thickness and the ratio of the leading edge shock wave thickness to the distance downstream from the leading edge. To a first approximation, the parameter is independent of the temperature conditions at the wall.

### 1. Introduction

IN the past few years a number of experimental investigations<sup>1-4</sup> have been made of the pressure and heat transfer distributions near the leading edge of a sharp flat plate in hypersonic, low Reynolds number flow. These investigations have shown that, as the leading edge is approached, both the surface pressure and heat transfer depart from the predictions of viscous interaction theory and tend toward constant "plateau" values.<sup>†</sup> It can be argued with some force that this leveling-off of the pressure and heat transfer is a slip-flow or rarefaction effect. Presumably, then, the departure of the data from the predictions of no-slip viscous interaction theory should be capable of correlation with some appropriately defined Knudsen number. Because of the strong variations in local mean free path across the shock wave and boundary layer, several quite different Knudsen numbers can be defined. The purpose here is to discuss some of these Knudsen numbers and to show that a particular one can be chosen which appears to correlate quite well the initial departure of the data from the predictions of no-slip viscous interaction theory.

### 2. Knudsen Number

The Knudsen number  $Kn$  is defined as the ratio  $\lambda/d$ , where  $\lambda$  is a relevant mean free path and  $d$  some dimension characteristic of the flow field. One can express  $Kn$  in terms of the local Mach and Reynolds numbers by use of the approximate kinetic theory relation

$$\lambda = (\mu/\rho a)(\pi\gamma/2)^{1/2} \quad (1)$$

in which  $\mu$  and  $\rho$  are the viscosity and density,  $\gamma$  the specific heats ratio, and  $a = (\gamma RT)^{1/2}$  the sound speed. One obtains the well-known formula

$$Kn = 1.25 \gamma^{1/2} M / Re_d \quad (2)$$

where  $M = U/a$ ,  $Re_d = \rho Ud/\mu$ , and  $U$  is some characteristic velocity.

In low-speed boundary layer flows, the characteristic flow field dimension  $d$  usually is taken to be the boundary layer displacement thickness  $\delta^*$ . Since the low-speed slip-flow

Received by ARS October 15, 1962; revision received February 20, 1963. This work was supported by Vidyia Inc., Palo Alto, Calif.

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† The region very close to the leading edge (within a few mean free paths at the wall), where the heat transfer and surface pressure behavior is not yet well established, is not included here.

regime is in essence a continuum flow perturbed by the alteration of the wall boundary condition, the incompressible boundary layer displacement thickness still is given approximately by  $\delta^*/x \sim (Re_{x_\infty})^{-1/2} = (\rho_\infty U_\infty x / \mu_\infty)^{-1/2}$ , with  $x$  the distance from the leading edge and infinity subscripts denoting freestream conditions. Thus, the Knudsen number  $\delta^*/\lambda_\infty$  for low-speed flow is given by

$$\lambda_\infty / \delta^* \simeq M_\infty / Re_{x_\infty}^{1/2} \quad (3)$$

omitting a numerical factor of order unity. This parameter  $M_\infty / Re_{x_\infty}^{1/2}$  is the one customarily used to define the slip-flow regime, and the limits on this regime are sometimes proposed to be

$$0.01 < M_\infty / Re_{x_\infty}^{1/2} < 0.1$$

The parameter  $M_\infty / Re_{x_\infty}^{1/2}$  has been used often as the slip parameter in the hypersonic regime as well. At first glance this would seem to be incorrect, since the most relevant mean free path is more likely to be the one at the wall, or at least behind the shock wave, and the hypersonic boundary layer displacement thickness does not vary as  $x Re_{x_\infty}^{-1/2}$ . Nevertheless, as will be seen, there is some justification for the use of the slip parameter  $M_\infty / Re_{x_\infty}^{1/2}$  even in the hypersonic regime.

### 3. Hypersonic Flat Plate Flow

It is by now well known that the effects that initially predominate in hypersonic flow over a sharp-edged flat plate as the gas density is lowered are viscous interaction phenomena. These phenomena, which are low Reynolds number effects associated with the mutual interaction between shock wave, boundary layer, and inviscid external flow, can be explained satisfactorily without introducing slip or temperature jump considerations into viscous interaction theory. For this reason the point of view is taken here that slip-flow effects in a hypersonic flat plate boundary layer, if they can in fact be identified, should be manifested as departures from the predictions of hypersonic viscous interaction theory. An excellent account of viscous interaction theory is given in Ref. 5, and the reader is referred to this source for more details of the theory to be discussed below. Hypersonic viscous interactions are governed by the interaction parameter  $\bar{\chi}_\infty = M_\infty^3 / \bar{Re}_{x_\infty}^{1/2}$ , where  $\bar{Re}_{x_\infty} = \rho_\infty U_\infty x / \mu_\infty C_\infty$ , and  $C_\infty = (\mu T_\infty) / (\mu_\infty T)$  is the Chapman-Rubens factor. The "strong interaction" region of the hypersonic boundary layer near the leading edge, characterized by  $\bar{\chi}_\infty \gg 1$ , is the region of concern, since slip effects can be expected to appear here first. It therefore is of interest to see what Knudsen numbers can be formed using consistent strong-interaction estimates for the flow in the neighborhood of the leading edge.

For any reference state (subscript  $r$ ), one can write

$$\frac{\lambda_r}{x} = \left( \frac{\pi\gamma}{2} \right)^{1/2} \left( \frac{\mu_r}{\rho_r a_r x} \right) = \left( \frac{\pi\gamma}{2} \right)^{1/2} \left( \frac{\mu_r}{\mu_\infty} \right) \left( \frac{\rho_\infty}{\rho_r} \right) \left( \frac{a_\infty}{a_r} \right) \frac{M_\infty}{Re_{x_\infty}} \quad (4)$$

Now, consistent with strong interaction theory, take  $(\mu_r / \mu_\infty) = C_\infty (T_r / T_\infty)$ . Also, for first-order strong interaction theory the surface pressure  $p_b$  is given by  $p_b = p_0 \bar{\chi}_\infty$ , and the displacement thickness of the boundary layer is given by  $\delta^* = x \delta_0 \bar{\chi}_\infty^{1/2} / M_\infty$ , where  $p_0$  and  $\delta_0$  are constants that depend on the wall temperature and that are related to one another by (Ref. 5, Sec. 9.3)

$$p_0 = \frac{9}{32} \gamma (\gamma + 1) \delta_0^2 \quad (5)$$

Inserting all of these into Eq. (4), and using  $(\rho_\infty / \rho_r) = (\rho_\infty T_r / \rho_r T_\infty)$ , one obtains

$$\frac{\lambda_r}{\delta^*} = \left( \frac{\pi\gamma}{2} \right)^{1/2} \left( \frac{T_r}{T_\infty} \right)^{3/2} \frac{M_\infty^2}{p_0 \delta_0 \bar{\chi}_\infty^{3/2} \bar{Re}_{x_\infty}} \quad (6)$$

At this point, it must be decided where the reference state is to be taken. If one looks for the maximum value of  $\lambda_r$ , it is reasonable to take, for Prandtl number equal to unity,  $(T_r / T_\infty) \cong (\gamma - 1) M_\infty^2 / 2$ , even for a cooled wall, since somewhere in the boundary layer the temperature may approach the recovery temperature. This gives

$$\frac{\lambda_{\max}}{\delta^*} = \frac{[(\pi\gamma)(\gamma + 1)^3]^{1/2} \bar{\chi}_\infty^{1/2}}{4 p_0 \delta_0 M_\infty} \quad (7)$$

On the other hand, if one takes  $T_r / T_\infty \simeq 1$  (supposing a highly cooled wall, with  $T_b \simeq T_r$ ), then one finds

$$\left( \frac{\lambda_b}{\delta^*} \right)_{\text{cold wall}} \cong \frac{(\pi\gamma/2)^{1/2} \bar{\chi}_\infty^{1/2}}{p_0 \delta_0 M_\infty^4} \quad (8)$$

which has a Mach number dependency quite different from Eq. (7). In Eq. (7) the cold-wall Knudsen number (at given values of  $\bar{\chi}_\infty$  and  $M_\infty$ ) exceeds the adiabatic-wall Knudsen number, because both  $p_0$  and  $\delta_0$  decrease with decreasing wall temperature. For the adiabatic wall, with  $\gamma = 1.4$ ,  $\delta_0 = 0.738$ , whereas for the highly cooled wall  $\delta_0 = 0.397$ .

A very interesting Knudsen number is obtained if one takes the reference condition as that behind the leading edge oblique shock,  $\lambda_r = \lambda_*$ . From oblique shock relations for large  $\bar{\chi}_\infty$ ,

$$\frac{T_s}{T_\infty} \cong \left( \frac{\gamma - 1}{\gamma + 1} \right) \frac{p_s}{p_\infty} \cong \left( \frac{\gamma - 1}{\gamma + 1} \right) p_0 \bar{\chi}_\infty \quad (9)$$

and, when this is inserted in Eq. (6), one finds

$$\lambda_s / \delta^* \cong F(\gamma) (M_\infty^2 / \bar{Re}_{x_\infty}) \quad (10)$$

where

$$F(\gamma) = \gamma \left( \frac{\gamma - 1}{\gamma + 1} \right)^{3/2} \left[ \frac{9\pi}{64} (\gamma + 1) \right]^{1/2} \quad (11)$$

$$[F(\gamma) \approx 0.1 \text{ for } \gamma = 1.4]$$

It is seen that  $\lambda_s / \delta^*$  contains neither  $p_0$  nor  $\delta_0$  but depends only on  $\gamma$  and the freestream conditions. Thus, to the present approximation, the Knudsen number  $\lambda_s / \delta^*$  should have the same value whether the wall is adiabatic or cooled. Also, one observes that  $\lambda_s / \delta^*$  involves simply the square of the low-speed rarefaction parameter  $M_\infty / Re_{x_\infty}^{1/2}$  (apart from the factor  $C_\infty$  that is contained in  $\bar{Re}_{x_\infty}$  and that would be unity in a low-speed constant property flow), so that, if the Knudsen number  $\lambda_s / \delta^*$  is a meaningful one, it indicates that the same rarefaction parameter  $M_\infty / Re_{x_\infty}^{1/2}$  should be applicable to both low-speed and hypersonic flows.

Another instructive and interesting interpretation may be given to the quantity  $M_\infty^2 / \bar{Re}_{x_\infty}$ , in terms of the leading-edge shock wave thickness. According to Mott-Smith theory, for example,<sup>6</sup> the maximum slope thickness  $\delta_m$  of a strong shock is given approximately by  $\delta_m / l^* \approx 5$ , where  $l^*$  is the shock center mean free path. (Other shock structure theories give results of the same order of magnitude.) But since the shock center is very nearly the sonic point, one may write  $l^* / x \approx M^* / Re_x^* \approx 1 / Re_x^*$ .<sup>†</sup> Again employing the Chapman-Rubens form of the linear viscosity-temperature law, and taking for a strong oblique shock  $\rho_\infty u_\infty = \rho^* u^*$ , one finds

$$\rho^* u^* x / \mu^* = Re_x^* \approx 2 \bar{Re}_{x_\infty} / (\gamma - 1) M_\infty^2 \quad (12)$$

for high Mach number. Hence,

$$\delta_m / x = \delta_m l^* / l^* x \approx M_\infty^2 / \bar{Re}_{x_\infty} \quad (13)$$

and one sees that the ratio of the maximum slope shock thickness to the distance from the leading edge is within a factor

<sup>†</sup> These estimates admittedly are rather crude and apply only where the shock is sufficiently strong that it can be treated as nearly normal.

of order unity given by the ratio  $M_\infty^2/\overline{Re}_{x_\infty}$ , thus providing an alternative interpretation for this rarefaction parameter.

#### 4. Comparison with Experiment

Perhaps the most interesting property of the Knudsen number  $\lambda_s/\delta^* = F(\gamma)M_\infty^2/\overline{Re}_{x_\infty}$  is that it is to first order independent of the temperature of the wall. It follows, therefore, that, if this Knudsen number is an appropriate one for correlating the first appearances of slip effects in the vicinity of the leading edge, the correlation should apply both to the cold-wall flows obtained in shock tunnels and the adiabatic-wall flows obtained in unheated low-density wind tunnels, and that the correlation should apply to both surface pressures and heat transfer. As will be seen, the existing experimental data support this conjecture.

The first observations of slip effects near the leading edge were obtained by Schaaf et al.<sup>1</sup> in their measurements of surface pressure distributions. These measurements were obtained in the unheated Berkeley low-density wind tunnel, at Mach numbers ranging from 3.7 to 5.8, with adiabatic wall conditions. Within the scatter of these data, one may estimate that the first significant departures from strong interaction theory occur at values of around  $M_\infty/(\overline{Re}_{x_\infty})^{1/2}$  between 0.2 and 0.3. More recently, Vidal and Wittliff<sup>4</sup> have reported surface pressure and heat transfer measurements obtained in the Cornell Aeronautical Laboratory low-density shock tunnel. These tests were in the Mach number range  $14 < M_\infty < 22$  and under cold-wall conditions where the wall temperature/stagnation temperature ratio was in the range  $0.07 < T_b/T_0 < 0.10$ . These authors, in fact, show that the departures of their data from interaction theory could be correlated with the parameter  $M_\infty/(\overline{Re}_{x_\infty})^{1/2}$ . They found that the departures from strong interaction theory occurred at about  $M_\infty/(\overline{Re}_{x_\infty})^{1/2} \sim 0.3$  for the heat transfer, and at about  $M_\infty/(\overline{Re}_{x_\infty})^{1/2} \sim 0.1$  for the surface pressure. It is significant that these values agree, probably within the accuracy of the several experiments, with the Berkeley data. The agreement is surprisingly good, when one considers the quite different flow conditions involved. Slip effects also have been observed by Nagumatsu et al.,<sup>2,3</sup> but unfortunately these data have a little too much scatter to permit accurate estimates of the value of  $M_\infty/(\overline{Re}_{x_\infty})^{1/2}$  at which slip effects appear.

To summarize, it appears that the parameter  $M_\infty/(\overline{Re}_{x_\infty})^{1/2}$  is appropriate for correlating the emergence of slip-flow effects near the leading edge of a flat plate in hypersonic flow. According to the first interpretation that has been given to this parameter, in terms of the Knudsen number  $\lambda_s/\delta^*$ , slip effects become important at about  $\lambda_s/\delta^* \sim 0.01$ , which is also the value of  $(\lambda/\delta^*)_{\text{low speed}}$  usually cited for the beginning of the low-speed slip-flow regime. According to the alternative interpretation of the parameter  $M_\infty/(\overline{Re}_{x_\infty})^{1/2}$  in terms of the  $\delta_m/x$ , slip effects appear when distance from the leading edge is less than roughly 10 times the shock thickness. Another demonstration is seen here of the fact that, when slip effects become important in the leading edge region, the shock wave no longer can be treated as a discontinuity.

#### References

- Schaaf, S. A., Hurlbut, F. C., Talbot, L., and Aroesty, J., "Viscous interaction experiments at low Reynolds numbers," ARS J. 29, 527-528 (1959).
- Nagamatsu, H. T., Sheer, R. E., and Schmid, J. R., "High temperature rarefied hypersonic flow over a flat plate," ARS J. 31, 902-910 (1961).
- Nagamatsu, H. T., Weil, J. A., and Sheer, R. E., Jr., "Heat transfer to flat plate in high temperature rarefied ultrahigh Mach number flow," ARS J. 32, 533-541 (1962).
- Vidal, R. J. and Wittliff, C. E., "Hypersonic low-density studies of blunt and slender bodies," *Proceedings of the Third*

*International Symposium on Rarefied Gas Dynamics*, edited by J. A. Laurmann (Academic Press Inc., New York, to be published).

<sup>5</sup> Hayes, W. D. and Probstein, R. F., *Hypersonic Flow Theory* (Academic Press Inc., New York, 1959), Chap. 9.

<sup>6</sup> Talbot, L., "Survey of the shock structure problem," ARS J. 32, 1009-1016 (1962).

## Effects of Winds Aloft on Loads of the Dyna-Soar Booster

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**T**HIS note presents the effects of winds aloft on the loads of a preliminary configuration Dyna-Soar booster, which was a modified Titan II missile with a glider at the forward end and large stabilizing fins at the aft end. The addition of the winged glider and fins provided large aerodynamic lifting surfaces that induced loads in the booster, under the influence of winds aloft.

Figure 1 shows the modified AviDyne 1% risk winds<sup>1</sup> used in the preliminary design of the booster. The maximum 1% envelope is used in conjunction with the 3000-ft wind shear data to construct discrete windspeed profiles similar to the ones illustrated for 25,000-, 30,000-, and 35,000-ft altitudes. The discrete wind profile to be applied, then, consists of the minimum windspeed curve, up to an altitude 3000 ft below the altitudes of interest, a 3000-ft wind shear to the maximum 1% envelope, a shear reversal of 3000 ft back to the minimum curve, and the minimum curve for the balance of the altitude range.

The discrete profiles shown in Fig. 1 are valid only for tailwinds. For application of other winds, such as sidewinds and headwinds, Fig. 2 illustrates the normalized wind rose,<sup>2</sup> which relates wind magnitude with direction. The wind rose is considered constant with altitude and is based on a tailwind for an easterly launch azimuth. Therefore, discrete windspeed profiles can be constructed for any wind direction by applying the wind rose of Fig. 2 to the maximum 1% envelope of Fig. 1. The wind shear data of Fig. 1 are considered applicable for any wind direction.

The relative bending moments of Fig. 3 show the bending moments associated with analytical flights through the discrete windspeed profiles of Fig. 1. Two different load analyses considered the response of the vehicle to a 3000-ft wind shear reversal at an altitude of 35,000 ft. (This altitude was found to be representatively critical for inducing lateral loads of a vibratory nature.) The dynamic analysis considered five degrees of freedom: rigid body rotation, rigid body translation, and the first three structural modes. In addition, consideration in this analysis was given to the dynamic characteristics of the autopilot and control systems. The pseudostatic load analysis essentially set all modal velocity and acceleration terms to zero. In doing this, velocities and accelerations were considered only in the rigid body modes of the vehicle.

A comparison of the data of Fig. 3 illustrates the adequacy of the pseudostatic analysis. Although the maximum dynamic bending moment did not occur at the point of windspeed shear reversal, as was the case of the pseudostatic

Presented at the ARS Launch Vehicles: Structures and Materials Conference, Phoenix, Ariz., April 3-5, 1962; revision received January 4, 1963.

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